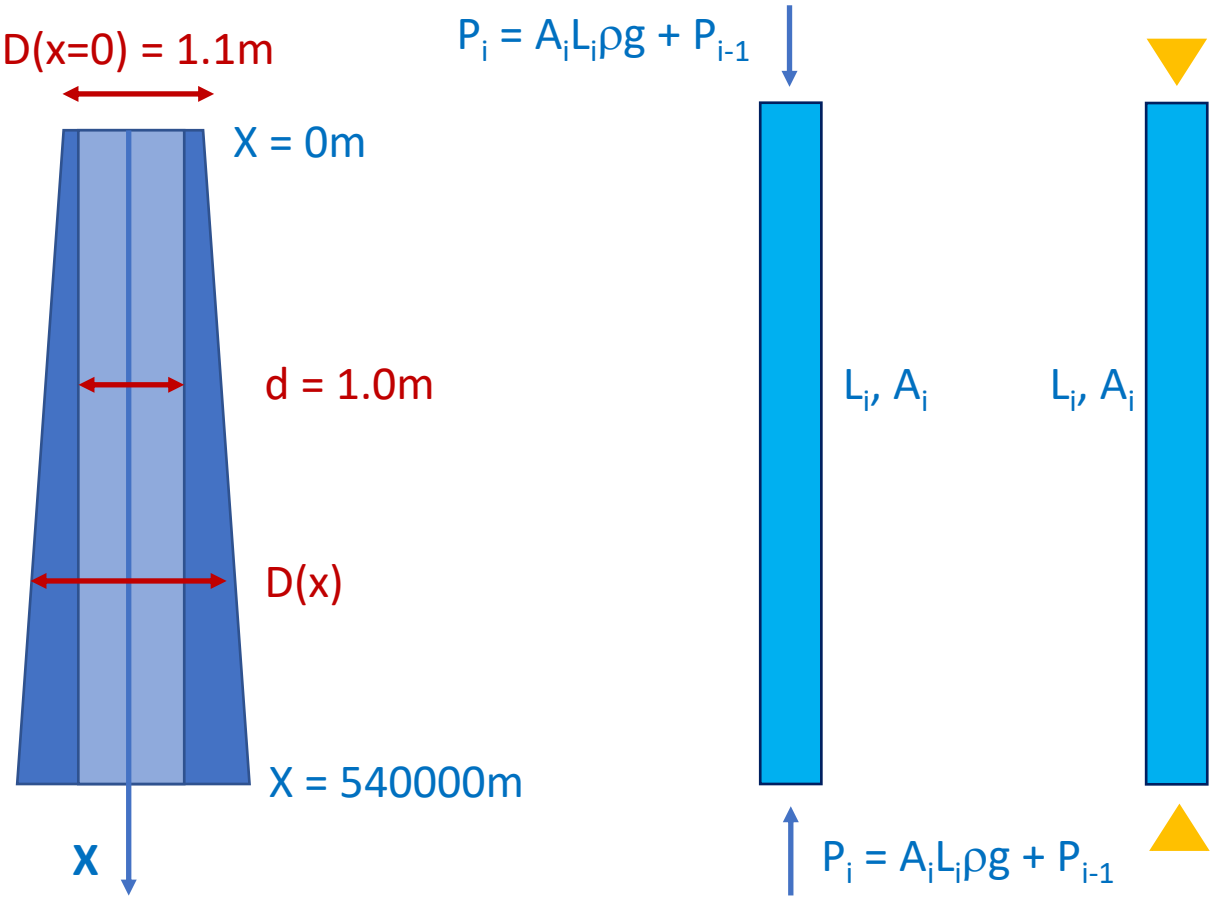
Superestrutura Umbilical Vertical capaz de resistir ao Peso Próprio Alessandro, 03/jan/2025

- Diâmetro Interno, d(x=qualquer) = d = 1.0m
- Diâmetro Externo, D(x=0m) = 1.1m, D(x) = variável a ser calculado para garantir a estabilidade estrutural
- Altura da Superestrutura Umbilical, H = 540000m (a razão dessa altura será explicada em outro documento)
- Material AL7574-T7351: densidade ρ = 2810 kg/m^3 , módulo de elasticidade E = 71700000 Pa, coeficiente de Poisson ν= 0.33, tensão de escoamento em compressão Fcy = 380000 Pa
- Aceleração da gravidade ao nível do mar (ligeiramente conservador para Superestrutura H), g = 9.807 m/s^2



Cálculo Iterativo da Estabilidade Estrutural

- Será considerada uma aproximação (modelo) para o cálculo iterativo da Estabilidade Estrutural, sendo integrada de x = 0m até x = 540000m = H, a Altura da Superestrutura Umbilical.
- Viga de diâmetro externo constante D_i , devendo suportar o seu peso próprio $A_iL_i\rho g$ mais o peso próprio de todas as seções constantes anteriores P_{i-1} ; logo: $P_i = A_iL_i\rho g + P_{i-1}$
- Condição de contorno rótula-rótula nas extremidades, cuja carga crítica é dada por

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

Superestrutura Umbilical Vertical capaz de resistir ao Peso Próprio

Alessandro, 03/jan/2025

Material =	AL70750-T7451	g =	9,807 m/s²
E =	71700000 Pa	L =	540000 m
Fcy =	380000 Pa	d =	1 m
rho =	2810 kg/m³	D0 =	1,1 m
Lmax =	14,12 m		

from Self-Buckling, wiki, english

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

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Self-buckling

1 language

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From Wikipedia, the free encyclopedia

A column can buckle due to its own weight with no other direct forces acting on it, in a failure mode called **self-buckling**. In conventional column buckling problems, the self-weight is often neglected since it is assumed to be small when compared to the applied axial loads. However, when this assumption is not valid, it is important to take the self-buckling into account.

Elastic buckling of a "heavy" column i.e., column buckling under its own weight, was first investigated by Greenhill in 1881.[1] He found that a free-standing, vertical column, with density ρ, Young's modulus E, and cross-sectional area A, will buckle under its own weight if its height exceeds a certain critical value:

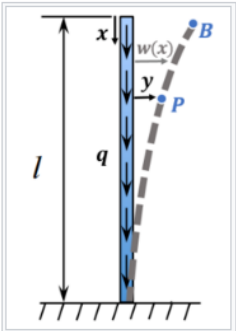
$$l_{\max} \approx \left(7.8373 \frac{EI}{\rho g A}\right)^{\frac{1}{3}}$$

where g is the acceleration due to gravity, I is the second moment of area of the beam cross section.

One interesting example for the use of the equation was suggested by Greenhill in his paper. He estimated the maximal height of a pine tree, and found it cannot grow over 300-ft tall. This length sets the maximum height for trees on earth if we assume the trees to be prismatic and the branches are neglected.

Mathematical derivation [edit]

Suppose a uniform column fixed in a vertical direction at its lowest point, and carried to a height l, in which the vertical position becomes unstable and flexure begins. There is a body force q per unit length $q = \rho g A$, where A is the cross-sectional area of the column, g is the acceleration due to gravity and ρ is its mass density.



A column exhibiting a compressive buckling load due to its own weight.

The column is slightly curved under its own weight, so the curve $w(x)$ describes the deflection of the beam in the y direction at some position x. Looking at any point on the column, we can write the moment equilibrium:

$$M = -\int_0^x q(y-w)\mathrm{d}x$$

where the right-hand side of the equation is the moment of the weight of BP about P.

According to Euler–Bernoulli beam theory:

$$M = -EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

Where E is the Young's modulus of elasticity of the substance, I is the second moment of area.

Therefore, the differential equation of the central line of BP is:

$$EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = q\int_0^x (y-w)\mathrm{d}x$$

Differentiating with respect to x, we get

$$EI\frac{\mathrm{d}^3 w}{\mathrm{d}x^3} = q\int_0^x \left(-\frac{\mathrm{d}w}{\mathrm{d}x}\right)\mathrm{d}x = -qx\frac{\mathrm{d}w}{\mathrm{d}x}$$

We get that the governing equation is the third order linear differential equation with a variable coefficient. The way to solve the problem is to use new variables n, z, k and r:

$$k^2 = \frac{4}{9}\frac{q}{EI},\quad r^2 = x^3,\quad \sqrt{x}z = \frac{\mathrm{d}w}{\mathrm{d}x},\quad n^2 = \frac{1}{3^2}$$

Then, the equation transforms to the Bessel equation

$$r^2\frac{\mathrm{d}^2 z}{\mathrm{d}r^2} + r\frac{\mathrm{d}z}{\mathrm{d}r} + \left(k^2 r^2 - n^2\right) z = 0$$

The solution of the transformed equation is $z = AJ_{\frac{1}{3}}(kr) + BJ_{-\frac{1}{3}}(kr)$

Where J_n is the Bessel function of the first kind. Then, the solution of the original equation is:

$$\frac{\mathrm{d}w}{\mathrm{d}x} = \sqrt{x}\left(AJ_{\frac{1}{3}}\left(kx^{\frac{3}{2}}\right) + BJ_{-\frac{1}{3},1}\left(kx^{\frac{3}{2}}\right)\right)$$

Now, we will use the boundary conditions:

- No moment at $x = 0 \rightarrow \frac{\mathrm{d}^2 w}{\mathrm{d}x^2}(x=0) = 0 \rightarrow A = 0$
- Fixed at $x = l \rightarrow w(l) = 0 \rightarrow J_{-\frac{1}{3}}\left(kl^{\frac{3}{2}}\right) = 0$

From the second B.C., we get that the critical length in which a vertical column will buckle under its own weight is:

$$l_{\max} = \left(\frac{j_{\frac{1}{3},1}}{k}\right)^{\frac{2}{3}} = \left(\frac{9j_{\frac{1}{3},1}^2}{4}\frac{EI}{q}\right)^{\frac{1}{3}}$$

Using $j_{\frac{1}{3},1} \approx 1.86635$, the first zero of the Bessel function of the first kind of order $-\frac{1}{3}$, $l_{\max}$ can be approximated to:

$$l_{\max} \approx \left(7.8373\frac{EI}{\rho g A}\right)^{\frac{1}{3}}$$

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Material =	AL70750-T7451	g =	9,807 m/s^2
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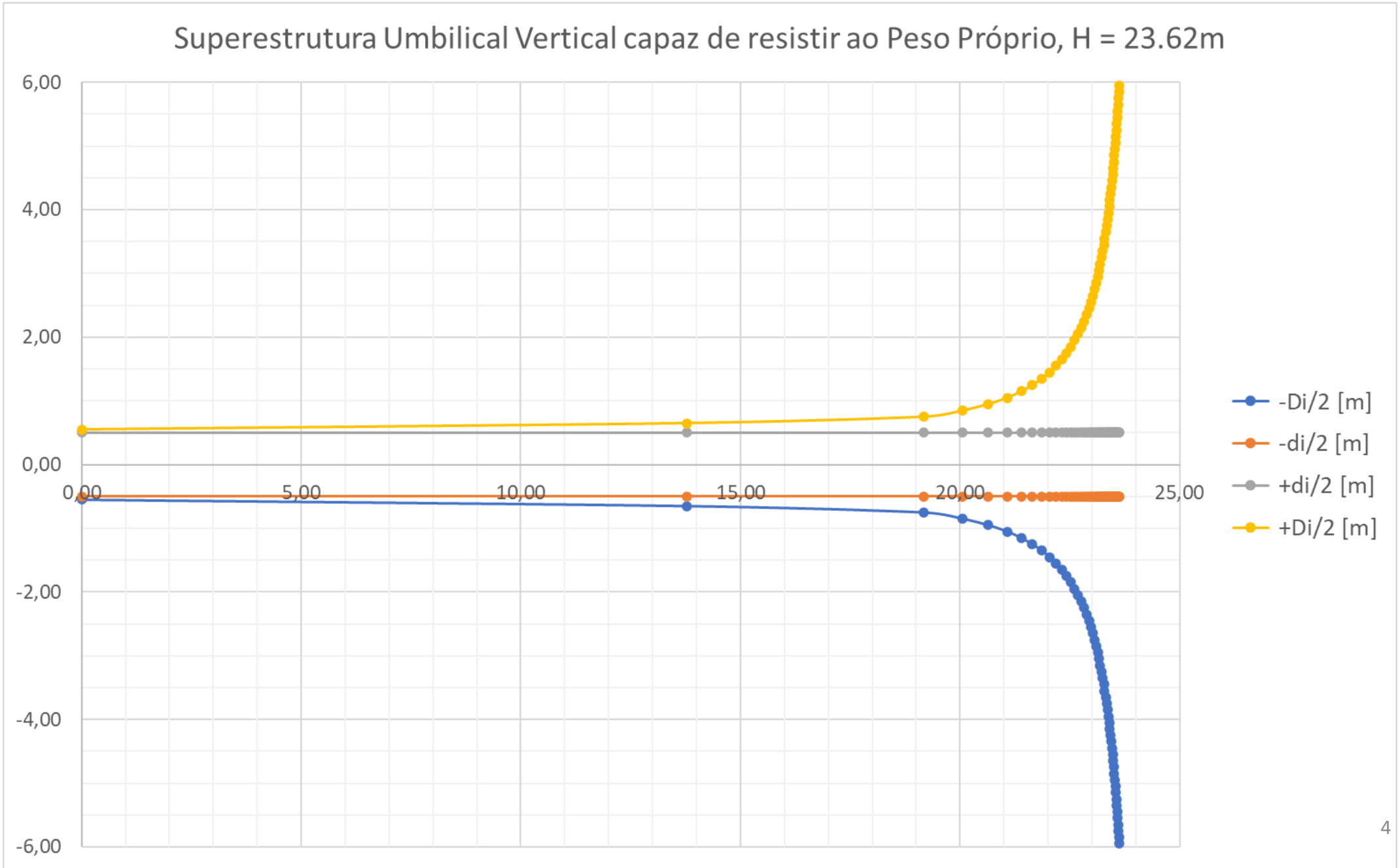
Lmax = 14,12 m
from Self-Buckling, wiki, english

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

Massa Total =	143,7 t = 1000x kg
Comprimento Total =	23,62 m
Diâmetro na Base =	11,90 m

iteração	d [m]	Di [m]	Ai [m^2]	Ei [m^4]	L_i [m]	m_i [kg]	P_i [N]	$Fcrit_i < Fcy$	$Pcrit_i$ [N]	P_i / $Pcrit_i$	x [m]	$-D_i/2$ [m]	$-d_i/2$ [m]	$+d_i/2$ [m]	$+D_i/2$ [m]
0	1,00	1,10	1,6493E-01	2,2781E-02	13,78	6,3865E+03	6,2633E+04	379745	8,4899E+04	7,3773E-01	0,00	-0,55	-0,50	0,50	0,55
1	1,00	1,30	5,4192E-01	9,1111E-02	5,40	1,4610E+04	2,0591E+05	379960	2,2111E+06	9,3127E-02	13,78	-0,65	-0,50	0,50	0,65
2	1,00	1,50	9,8175E-01	1,9942E-01	0,88	1,7037E+04	3,7300E+05	379930	1,8223E+08	2,0469E-03	19,18	-0,75	-0,50	0,50	0,75
3	1,00	1,70	1,4844E+00	3,6090E-01	0,58	1,9457E+04	5,6381E+05	379821	7,5918E+08	7,4265E-04	20,06	-0,85	-0,50	0,50	0,85
4	1,00	1,90	2,0499E+00	5,9062E-01	0,43	2,1934E+04	7,7891E+05	379976	2,2604E+09	3,4458E-04	20,64	-0,95	-0,50	0,50	0,95
5	1,00	2,10	2,6782E+00	9,0557E-01	0,32	2,4342E+04	1,0176E+06	379966	6,2581E+09	1,6261E-04	21,07	-1,05	-0,50	0,50	1,05
6	1,00	2,30	3,3694E+00	1,3246E+00	0,25	2,6709E+04	1,2796E+06	379764	1,4997E+10	8,5319E-05	21,39	-1,15	-0,50	0,50	1,15
7	1,00	2,50	4,1233E+00	1,8684E+00	0,22	2,9258E+04	1,5665E+06	379908	2,7317E+10	5,7344E-05	21,64	-1,25	-0,50	0,50	1,25
8	1,00	2,70	4,9402E+00	2,5596E+00	0,17	3,1618E+04	1,8766E+06	379860	6,2675E+10	2,9941E-05	21,86	-1,35	-0,50	0,50	1,35
9	1,00	2,90	5,8198E+00	3,4228E+00	0,15	3,4071E+04	2,2107E+06	379858	1,0765E+11	2,0536E-05	22,03	-1,45	-0,50	0,50	1,45
10	1,00	3,10	6,7623E+00	4,4842E+00	0,13	3,6541E+04	2,5691E+06	379910	1,8777E+11	1,3682E-05	22,18	-1,55	-0,50	0,50	1,55
11	1,00	3,30	7,7676E+00	5,7723E+00	0,11	3,8942E+04	2,9510E+06	379907	3,3758E+11	8,7414E-06	22,31	-1,65	-0,50	0,50	1,65
12	1,00	3,50	8,8357E+00	7,3171E+00	0,10	4,1425E+04	3,3572E+06	379959	5,1779E+11	6,4837E-06	22,42	-1,75	-0,50	0,50	1,75

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CONCLUSÃO

- Infelizmente uma Superestrutura Umbilical Vertical capaz de resistir ao Peso Próprio, conforme concebida aqui e em material AL7475-T7451 jamais poderia ter uma altura H = 540000m = 540 km
- A estrutura umbilical vertical dimensionada neste estudo é apresentada no slide anterior e apresenta uma altura H = 23.62m, o seu diâmetro na base tem D = 11.9m, e o seu diâmetro no topo tem D = 1.1m e diâmetro interno constate d = 1.0m
- Esta estrutura lembra aquela do tronco de uma árvore; novamente as soluções otimizadas apresentando suas semelhanças com as estruturas observáveis na natureza
- Para saber mais sobre estruturas esbeltas capazes de resistir ao peso próprio, acesse https://en.wikipedia.org/wiki/Self-buckling
- Para saber mais sobre elevadores espaciais, o conceito que originou este estudo, acesse https://en.wikipedia.org/wiki/Space_elevator
- Li sobre a ideia no The Three-Body Problem, premiado livro de ficção científica de https://en.wikipedia.org/wiki/The_Three-Body_Problem_(novel)
- Algumas páginas inspiradoras são apresentadas nos próximos três slides
- Fique Bem \o/

* * *

The trial run of *Tianti III* was airing on television. Construction on three space elevators had begun five years ago, and since *Tianti I* and *Tianti II* had been put into operation at the start of the year, the test of *Tianti III* did not cause much of a commotion. All space elevators were currently being built with just a single primary rail, giving them a far smaller carrying capacity than the four-rail models still under design, but this was already an altogether different world from the age of chemical rockets. Setting aside construction, the cost of going into space by elevator was substantially lower than by civilian aircraft. This in turn had led to an increase in

the number of bodies in motion in Earth's night sky: These were humanity's large-scale orbiting structures.

Tianti III was the only space elevator based on the ocean. Its base was located on the Equator on an artificial floating island in the Pacific Ocean that could navigate at sea under its own nuclear power, which meant that the elevator's position on the Equator could be adjusted if necessary. The floating island was a real-life version of the Propeller Island Jules Verne had described, and so it had been dubbed "Verne Island." The ocean wasn't even visible on the television, which was showing a shot of a metal, pyramid-shaped base surrounded by a steel city, and—at the bottom of the rail—the cylindrical transport cabin that was ready to

launch. From this distance, the guide rail extending into space was invisible due to its sixty-centimeter diameter, although at times you could catch a glint of reflected light from the setting sun.

Three old men, Zhang Yuanchao and his two old neighbors, Yang Jinwen and Miao Fuquan, were watching this on television. All of them were now past seventy, and while no one would call them doddering, they were now definitely old. For them, recalling the past and looking toward the future were both burdens, and since they were powerless to do anything about the present, their only option was to live out their waning years without thinking about anything in this unusual era.