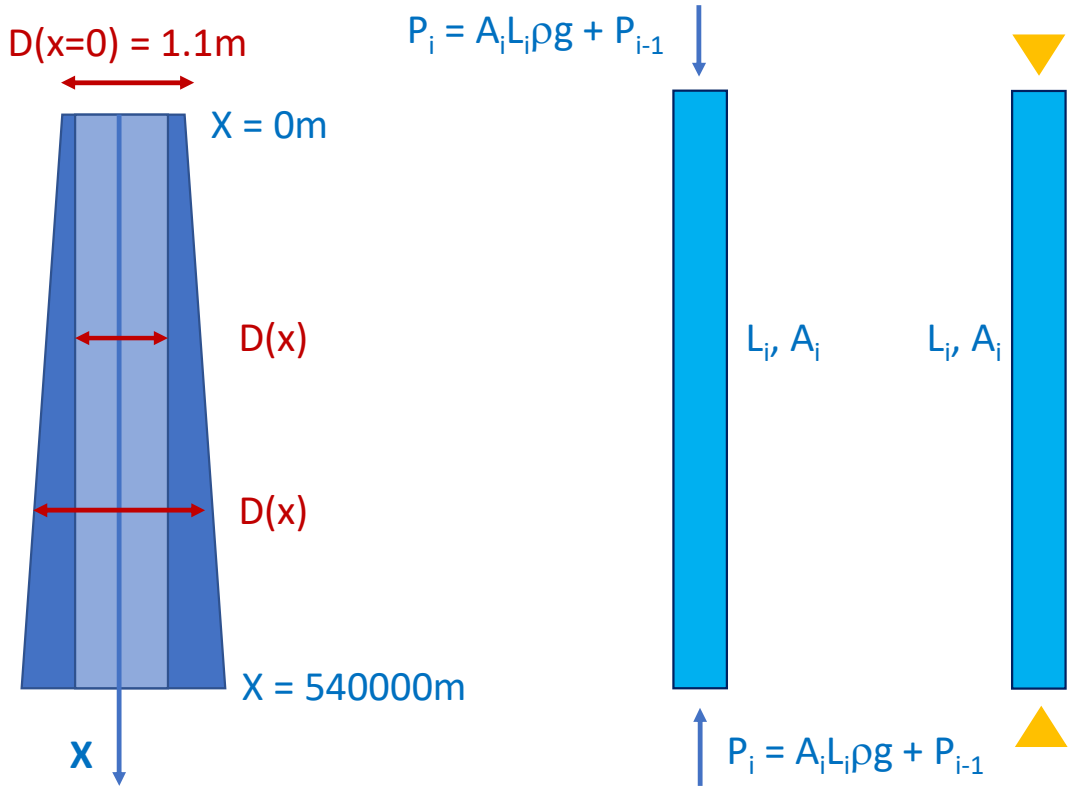
Superestrutura Elevador Espacial Xerxes (resiste ao Peso Próprio) Alessandro, 08/jan/2025

- Diâmetro Interno, d(x=0) = 1.0m, Assume-se D(x)/d(x) = 1.1
- Diâmetro Externo, D(x=0m) = 1.1m, D(x) = variável a ser calculado para garantir a estabilidade estrutural
- Altura da Superestrutura Umbilical, H = 540000m (a razão dessa altura será explicada em outro documento)
- Material AL7574-T7351: densidade ρ = 2810 kg/m^3 , módulo de elasticidade E = 71700000 Pa, coeficiente de Poisson ν= 0.33, tensão de escoamento em compressão Fcy = 380000 Pa
- Aceleração da gravidade ao nível do mar (ligeiramente conservador para Superestrutura H), g = 9.807 m/s^2



Cálculo Iterativo da Estabilidade Estrutural

- Será considerada uma aproximação (modelo) para o cálculo iterativo da Estabilidade Estrutural, sendo integrada de x = 0m até x = 540000m = H, a Altura da Superestrutura Umbilical.
- Viga de diâmetro externo constante D_i , devendo suportar o seu peso próprio $A_iL_i\rho g$ mais o peso próprio de todas as seções constantes anteriores P_{i-1} ; logo: $P_i = A_iL_i\rho g + P_{i-1}$
- Condição de contorno rótula-rótula nas extremidades, cuja carga crítica é dada por

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

Superestrutura Elevador Espacial Xerxes (resiste ao Peso Próprio)
Alessandro, 08/jan/2025

Material = AL70750-T7451			g =	9,807	m/s²
E =	71700000	Pa	L =	540000	m
Fcy =	380000	Pa	d =	1	m
rho =	2810	kg/m³	D0 =	1,1	m
Lmax =	14,122	m			
from Self-Buckling, wiki, english					

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

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Self-buckling

1 language

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From Wikipedia, the free encyclopedia

A column can buckle due to its own weight with no other direct forces acting on it, in a failure mode called **self-buckling**. In conventional column buckling problems, the self-weight is often neglected since it is assumed to be small when compared to the applied axial loads. However, when this assumption is not valid, it is important to take the self-buckling into account.

Elastic buckling of a "heavy" column i.e., column buckling under its own weight, was first investigated by Greenhill in 1881.[1] He found that a free-standing, vertical column, with density ρ, Young's modulus E, and cross-sectional area A, will buckle under its own weight if its height exceeds a certain critical value:

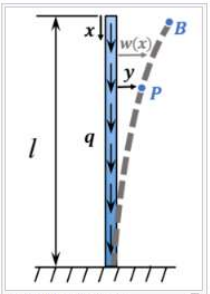
$$l_{\max} \approx \left(7.8373 \frac{EI}{\rho g A}\right)^{\frac{1}{3}}$$

where g is the acceleration due to gravity, I is the second moment of area of the beam cross section.

One interesting example for the use of the equation was suggested by Greenhill in his paper. He estimated the maximal height of a pine tree, and found it cannot grow over 300-ft tall. This length sets the maximum height for trees on earth if we assume the trees to be prismatic and the branches are neglected.

Mathematical derivation [edit]

Suppose a uniform column fixed in a vertical direction at its lowest point, and carried to a height l, in which the vertical position becomes unstable and flexure begins. There is a body force q per unit length $q = \rho g A$, where A is the cross-sectional area of the column, g is the acceleration due to gravity and ρ is its mass density.



A column exhibiting a compressive buckling load due to its own weight.

The column is slightly curved under its own weight, so the curve $w(x)$ describes the deflection of the beam in the y direction at some position x. Looking at any point on the column, we can write the moment equilibrium:

$$M = -\int_0^x q(y-w)\mathrm{d}x$$

where the right-hand side of the equation is the moment of the weight of BP about P.

According to Euler–Bernoulli beam theory:

$$M = -EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}$$

Where E is the Young's modulus of elasticity of the substance, I is the second moment of area.

Therefore, the differential equation of the central line of BP is:

$$EI\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = q\int_0^x (y-w)\mathrm{d}x$$

Differentiating with respect to x, we get

$$EI\frac{\mathrm{d}^3 w}{\mathrm{d}x^3} = q\int_0^x \left(-\frac{\mathrm{d}w}{\mathrm{d}x}\right)\mathrm{d}x = -qx\frac{\mathrm{d}w}{\mathrm{d}x}$$

We get that the governing equation is the third order linear differential equation with a variable coefficient. The way to solve the problem is to use new variables n, z, k and r:

$$k^2 = \frac{4}{9}\frac{q}{EI}, \quad r^2 = x^3, \quad \sqrt{x}z = \frac{\mathrm{d}w}{\mathrm{d}x}, \quad n^2 = \frac{1}{3^2}$$

Then, the equation transforms to the Bessel equation

$$r^2\frac{\mathrm{d}^2 z}{\mathrm{d}r^2} + r\frac{\mathrm{d}z}{\mathrm{d}r} + \left(k^2 r^2 - n^2\right) z = 0$$

The solution of the transformed equation is $z = AJ_{\frac{1}{3}}(kr) + BJ_{-\frac{1}{3}}(kr)$

Where J_n is the Bessel function of the first kind. Then, the solution of the original equation is:

$$\frac{\mathrm{d}w}{\mathrm{d}x} = \sqrt{x}\left(AJ_{\frac{1}{3}}\left(kx^{\frac{3}{2}}\right) + BJ_{-\frac{1}{3},1}\left(kx^{\frac{3}{2}}\right)\right)$$

Now, we will use the boundary conditions:

- No moment at $x = 0 \rightarrow \frac{\mathrm{d}^2 w}{\mathrm{d}x^2}(x=0) = 0 \rightarrow A = 0$
- Fixed at $x = l \rightarrow w(l) = 0 \rightarrow J_{-\frac{1}{3}}\left(kl^{\frac{3}{2}}\right) = 0$

From the second B.C., we get that the critical length in which a vertical column will buckle under its own weight is:

$$l_{\max} = \left(\frac{j_{\frac{1}{3},1}}{k}\right)^{\frac{2}{3}} = \left(\frac{9j_{\frac{1}{3},1}^2}{4}\frac{EI}{q}\right)^{\frac{1}{3}}$$

Using $j_{\frac{1}{3},1} \approx 1.86635$, the first zero of the Bessel function of the first kind of order $-\frac{1}{3}$, $l_{\max}$ can be approximated to:

$$l_{\max} \approx \left(7.8373\frac{EI}{\rho g A}\right)^{\frac{1}{3}}$$

Superestrutura Elevador Espacial Xerxes (resiste ao Peso Próprio) Alessandro, 08/jan/2025

Material =	AL70750-T7451		g =	9,807	m/s²
E =	71700000	Pa	L =	540000	m
Fcy =	380000	Pa	d =	1	m
rho =	2810	kg/m³	D0 =	1,1	m
Lmax =	14,122	m			

from Self-Buckling, wiki, english

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

Massa Total =	881,5	t = 1000x kg
Altura Total =	15,96	m
Diâmetro na Base =	30,91	m
H =	0,456	m
Taxa Crescimento d e D =	1,100	[-]

iteração	d [m]	Di [m]	Ai [m²]	Ii [m⁴]	L_i [m]	m_i [kg]	P_i [N]	$Fcrit_i < Fcy$	$Pcrit_i$ [N]	P_i / $Pcrit_i$	x [m]	$-D_i/2$ [m]	$-d_i/2$ [m]	$+d_i/2$ [m]	$+D_i/2$ [m]
1	1,0000	1,1000	1,6493E-01	2,2781E-02	0,456	2,4063E+02	2,3598E+03	14308	7,7506E+07	3,0447E-05	0,00	-0,55	-0,50	0,50	0,55
2	1,1000	1,2100	1,9957E-01	3,3354E-02	0,456	4,7525E+02	7,0206E+03	35179	1,1348E+08	6,1868E-05	0,46	-0,61	-0,55	0,55	0,61
3	1,2100	1,3310	2,4148E-01	4,8834E-02	0,456	7,5914E+02	1,4466E+04	59904	1,6614E+08	8,7068E-05	0,91	-0,67	-0,61	0,61	0,67
4	1,3310	1,4641	2,9219E-01	7,1498E-02	0,456	1,1027E+03	2,5279E+04	86516	2,4325E+08	1,0392E-04	1,37	-0,73	-0,67	0,67	0,73
5	1,4641	1,6105	3,5355E-01	1,0468E-01	0,456	1,5183E+03	4,0169E+04	113617	3,5614E+08	1,1279E-04	1,82	-0,81	-0,73	0,73	0,81
6	1,6105	1,7716	4,2780E-01	1,5326E-01	0,456	2,0212E+03	5,9992E+04	140234	5,2142E+08	1,1505E-04	2,28	-0,89	-0,81	0,81	0,89
7	1,7716	1,9487	5,1763E-01	2,2439E-01	0,456	2,6298E+03	8,5782E+04	165720	7,6341E+08	1,1237E-04	2,74	-0,97	-0,89	0,89	0,97
8	1,9487	2,1436	6,2634E-01	3,2853E-01	0,456	3,3661E+03	1,1879E+05	189665	1,1177E+09	1,0628E-04	3,19	-1,07	-0,97	0,97	1,07
9	2,1436	2,3579	7,5787E-01	4,8100E-01	0,456	4,2571E+03	1,6054E+05	211836	1,6364E+09	9,8105E-05	3,65	-1,18	-1,07	1,07	1,18
10	2,3579	2,5937	9,1702E-01	7,0424E-01	0,456	5,3352E+03	2,1287E+05	232129	2,3959E+09	8,8845E-05	4,10	-1,30	-1,18	1,18	1,30
11	2,5937	2,8531	1,1096E+00	1,0311E+00	0,456	6,6397E+03	2,7798E+05	250526	3,5079E+09	7,9245E-05	4,56	-1,43	-1,30	1,30	1,43
12	2,8531	3,1384	1,3426E+00	1,5096E+00	0,456	8,2181E+03	3,5858E+05	267075	5,1358E+09	6,9818E-05	5,02	-1,57	-1,43	1,43	1,57
13	3,1384	3,4523	1,6246E+00	2,2102E+00	0,456	1,0128E+04	4,5790E+05	281864	7,5194E+09	6,0896E-05	5,47	-1,73	-1,57	1,57	1,73
14	3,4523	3,7975	1,9657E+00	3,2359E+00	0,456	1,2439E+04	5,7989E+05	295004	1,1009E+10	5,2674E-05	5,93	-1,90	-1,73	1,73	1,90
15	3,7975	4,1772	2,3785E+00	4,7377E+00	0,456	1,5235E+04	7,2930E+05	306623	1,6118E+10	4,5246E-05	6,39	-2,09	-1,90	1,90	2,09
16	4,1772	4,5950	2,8780E+00	6,9365E+00	0,456	1,8619E+04	9,1190E+05	316852	2,3599E+10	3,8641E-05	6,84	-2,30	-2,09	2,09	2,30
17	4,5950	5,0545	3,4824E+00	1,0156E+01	0,456	2,2713E+04	1,1346E+06	325825	3,4551E+10	3,2839E-05	7,30	-2,53	-2,30	2,30	2,53
18	5,0545	5,5599	4,2137E+00	1,4869E+01	0,456	2,7667E+04	1,4060E+06	333669	5,0587E+10	2,7793E-05	7,75	-2,78	-2,53	2,53	2,78
19	5,5599	6,1159	5,0985E+00	2,1770E+01	0,456	3,3661E+04	1,7361E+06	340506	7,4064E+10	2,3440E-05	8,21	-3,06	-2,78	2,78	3,06
20	6,1159	6,7275	6,1692E+00	3,1873E+01	0,456	4,0914E+04	2,1373E+06	346448	1,0844E+11	1,9710E-05	8,67	-3,36	-3,06	3,06	3,36

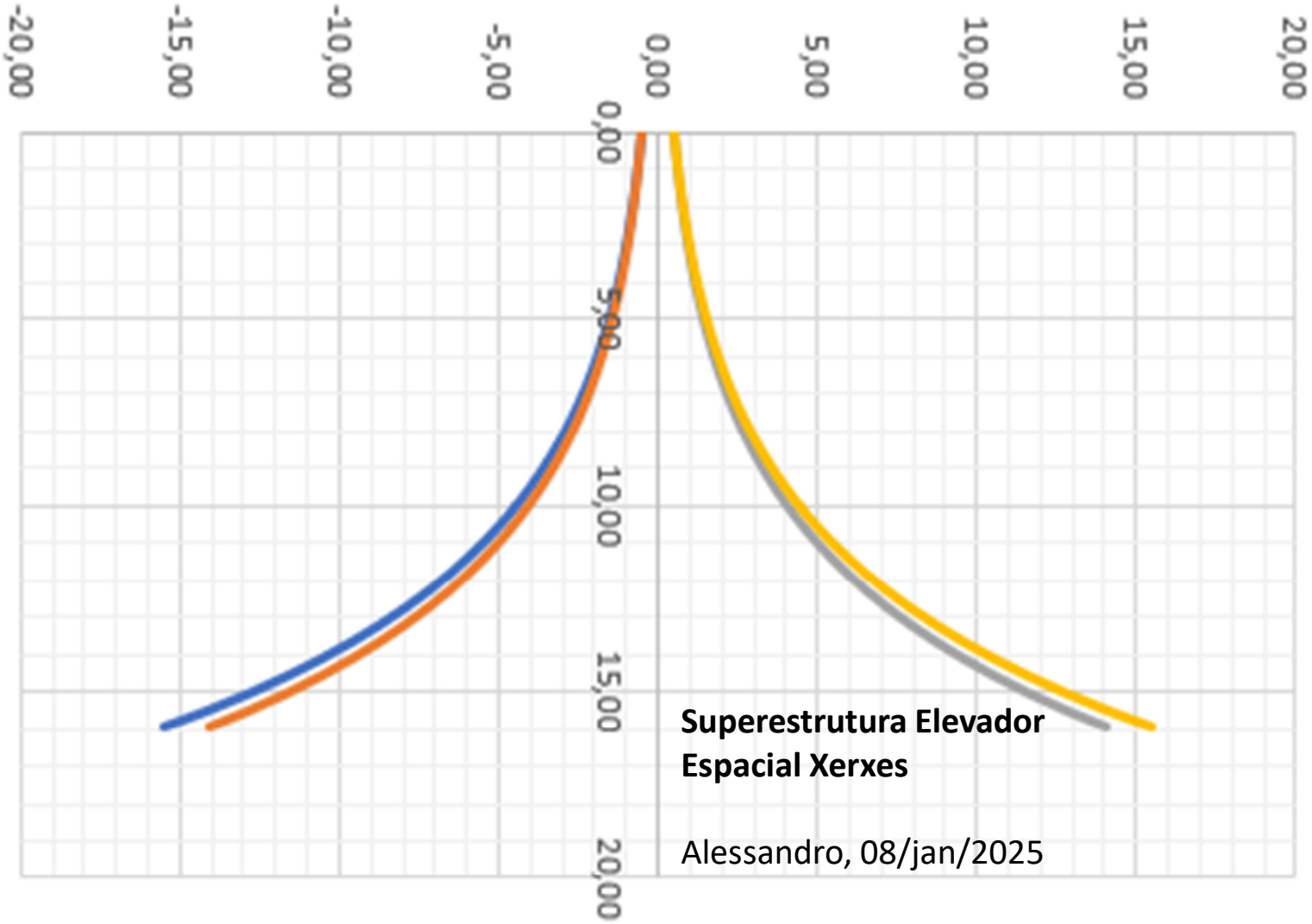
Superestrutura Elevador Espacial Xerxes (resiste ao Peso Próprio) Alessandro, 08/jan/2025

Material = AL70750-T7451			g =	9,807	m/s²
E =	71700000	Pa	L =	540000	m
Fcy =	380000	Pa	d =	1	m
rho =	2810	kg/m³	D0 =	1,1	m
Lmax =	14,122	m			
from Self-Buckling, wiki, english					

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

Massa Total =	881,5	t = 1000x kg
Altura Total =	15,96	m
Diâmetro na Base =	30,91	m
H =	0,456	m
Taxa Crescimento d e D =	1,100	[-]

18	5,0545	5,5599	4,2137E+00	1,4869E+01	0,456	2,7667E+04	1,4060E+06	333669	5,0587E+10	2,7793E-05	7,75	-2,78	-2,53	2,53	2,78
19	5,5599	6,1159	5,0985E+00	2,1770E+01	0,456	3,3661E+04	1,7361E+06	340506	7,4064E+10	2,3440E-05	8,21	-3,06	-2,78	2,78	3,06
20	6,1159	6,7275	6,1692E+00	3,1873E+01	0,456	4,0914E+04	2,1373E+06	346448	1,0844E+11	1,9710E-05	8,67	-3,36	-3,06	3,06	3,36
21	6,7275	7,4002	7,4648E+00	4,6666E+01	0,456	4,9690E+04	2,6246E+06	351602	1,5876E+11	1,6532E-05	9,12	-3,70	-3,36	3,36	3,70
22	7,4002	8,1403	9,0324E+00	6,8323E+01	0,456	6,0308E+04	3,2161E+06	356060	2,3244E+11	1,3836E-05	9,58	-4,07	-3,70	3,70	4,07
23	8,1403	8,9543	1,0929E+01	1,0003E+02	0,456	7,3157E+04	3,9335E+06	359910	3,4032E+11	1,1558E-05	10,03	-4,48	-4,07	4,07	4,48
24	8,9543	9,8497	1,3224E+01	1,4646E+02	0,456	8,8704E+04	4,8034E+06	363229	4,9827E+11	9,6403E-06	10,49	-4,92	-4,48	4,48	4,92
25	9,8497	10,8347	1,6001E+01	2,1443E+02	0,456	1,0752E+05	5,8579E+06	366084	7,2951E+11	8,0298E-06	10,95	-5,42	-4,92	4,92	5,42
26	10,8347	11,9182	1,9362E+01	3,1394E+02	0,456	1,3028E+05	7,1355E+06	368537	1,0681E+12	6,6807E-06	11,40	-5,96	-5,42	5,42	5,96
27	11,9182	13,1100	2,3428E+01	4,5964E+02	0,456	1,5782E+05	8,6833E+06	370641	1,5638E+12	5,5528E-06	11,86	-6,55	-5,96	5,96	6,55
28	13,1100	14,4210	2,8347E+01	6,7296E+02	0,456	1,9115E+05	1,0558E+07	372444	2,2895E+12	4,6114E-06	12,31	-7,21	-6,55	6,55	7,21
29	14,4210	15,8631	3,4300E+01	9,8529E+02	0,456	2,3147E+05	1,2828E+07	373987	3,3521E+12	3,8268E-06	12,77	-7,93	-7,21	7,21	7,93
30	15,8631	17,4494	4,1504E+01	1,4426E+03	0,456	2,8027E+05	1,5576E+07	375305	4,9078E+12	3,1738E-06	13,23	-8,72	-7,93	7,93	8,72
31	17,4494	19,1943	5,0219E+01	2,1120E+03	0,456	3,3931E+05	1,8904E+07	376431	7,1855E+12	2,6309E-06	13,68	-9,60	-8,72	8,72	9,60
32	19,1943	21,1138	6,0765E+01	3,0922E+03	0,456	4,1075E+05	2,2932E+07	377391	1,0520E+13	2,1798E-06	14,14	-10,56	-9,60	9,60	10,56
33	21,1138	23,2252	7,3526E+01	4,5274E+03	0,456	4,9719E+05	2,7808E+07	378208	1,5403E+13	1,8054E-06	14,59	-11,61	-10,56	10,56	11,61
34	23,2252	25,5477	8,8966E+01	6,6285E+03	0,456	6,0178E+05	3,3710E+07	378905	2,2551E+13	1,4948E-06	15,05	-12,77	-11,61	11,61	12,77
35	25,5477	28,1024	1,0765E+02	9,7048E+03	0,456	7,2834E+05	4,0853E+07	379497	3,3017E+13	1,2373E-06	15,51	-14,05	-12,77	12,77	14,05
36	28,1024	30,9127	1,3026E+02	1,4209E+04	0,456	8,8147E+05	4,9497E+07	380000	4,8340E+13	1,0239E-06	15,96	-15,46	-14,05	14,05	15,46



Superestrutura Cônica Vertical Peso Próprio: D/d =1.1, H = 15.96m, D = 30.91m, M =882t

Superestrutura Elevador Espacial Xerxes (resiste ao Peso Próprio) Alessandro, 08/jan/2025

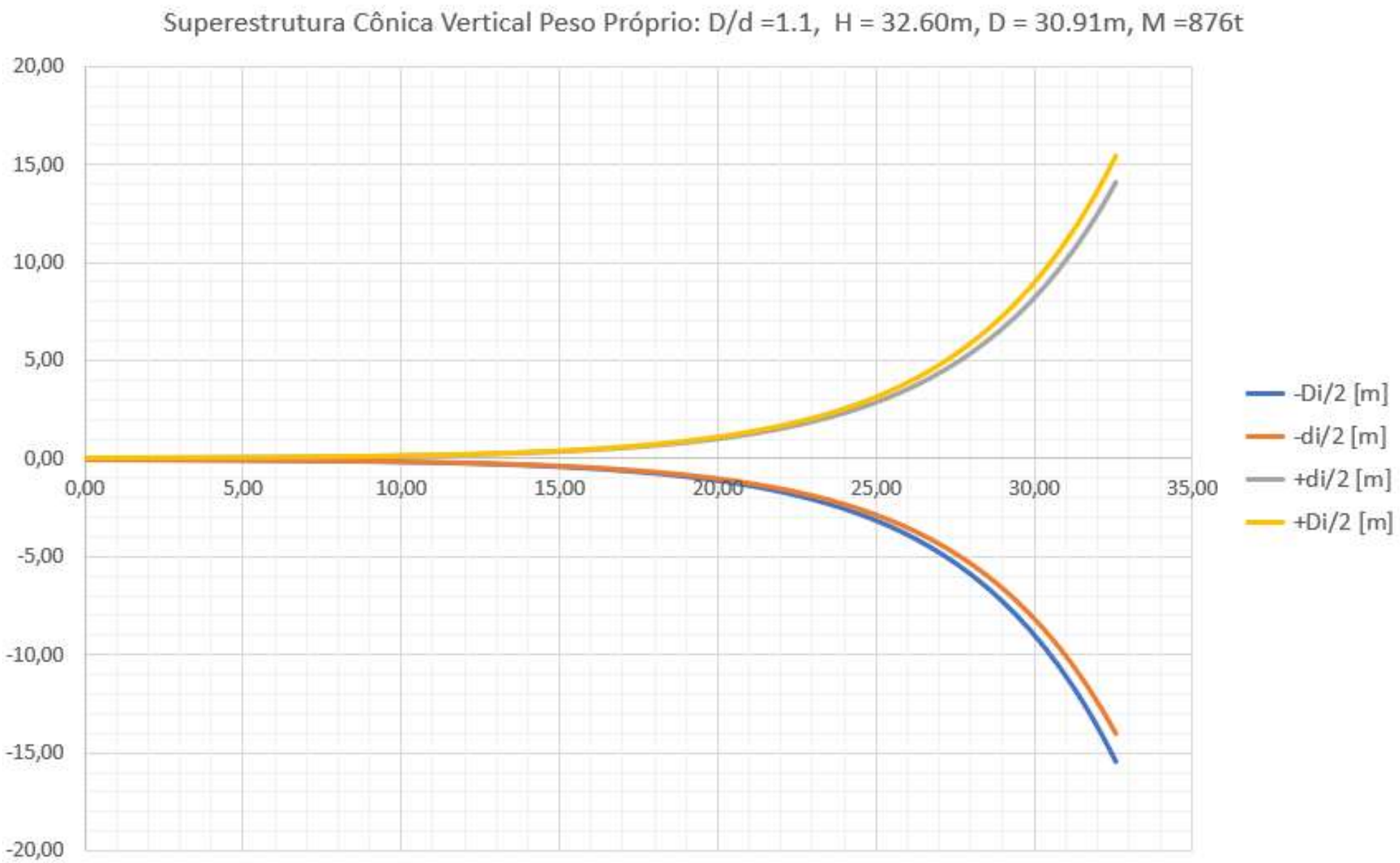
Material =	AL70750-T7451		g =	9,807	m/s²
E =	71700000	Pa	L =	540000	m
Fcy =	380000	Pa	d =	1	m
rho =	2810	kg/m³	D0 =	1,1	m

Lmax = 14,122 m
from Self-Buckling, wiki, english

$$p_{cr} = \pi^2 \frac{EI_i}{L_i^2}$$

Massa Total =	876,0	t = 1000x kg
Altura Total =	32,60	m
Diâmetro na Base =	30,91	m
H =	0,453	m
Taxa Crescimento d e D =	1,100	[-]

iteração	d [m]	Di [m]	Ai [m²]	Ii [m⁴]	L_i [m]	m_i [kg]	P_i [N]	$Fcrit_i$ < Fcy	$Pcrit_i$ [N]	P_i / $Pcrit_i$	x [m]	$-D_i/2$ [m]	$-d_i/2$ [m]	$+d_i/2$ [m]	$+D_i/2$ [m]
-36	0,0294	0,0323	1,4264E-04	1,7040E-08	0,453	2,0660E-01	2,0261E+00	14204	5,8821E+01	3,4446E-02	0,00	-0,02	-0,01	0,01	0,02
-35	0,0323	0,0356	1,7260E-04	2,4948E-08	0,453	4,0804E-01	6,0278E+00	34924	8,6119E+01	6,9993E-02	0,45	-0,02	-0,02	0,02	0,02
-34	0,0356	0,0391	2,0884E-04	3,6526E-08	0,453	6,5179E-01	1,2420E+01	59470	1,2609E+02	9,8502E-02	0,91	-0,02	-0,02	0,02	0,02
-33	0,0391	0,0431	2,5270E-04	5,3478E-08	0,453	9,4673E-01	2,1704E+01	85890	1,8460E+02	1,1757E-01	1,36	-0,02	-0,02	0,02	0,02
-32	0,0431	0,0474	3,0577E-04	7,8297E-08	0,453	1,3036E+00	3,4489E+01	112794	2,7028E+02	1,2760E-01	1,81	-0,02	-0,02	0,02	0,02
-31	0,0474	0,0521	3,6998E-04	1,1464E-07	0,453	1,7354E+00	5,1508E+01	139219	3,9572E+02	1,3016E-01	2,26	-0,03	-0,02	0,02	0,03
-30	0,0521	0,0573	4,4767E-04	1,6784E-07	0,453	2,2579E+00	7,3651E+01	164520	5,7937E+02	1,2712E-01	2,72	-0,03	-0,03	0,03	0,03
-29	0,0573	0,0630	5,4169E-04	2,4573E-07	0,453	2,8901E+00	1,0199E+02	188291	8,4825E+02	1,2024E-01	3,17	-0,03	-0,03	0,03	0,03
-28	0,0630	0,0693	6,5544E-04	3,5977E-07	0,453	3,6551E+00	1,3784E+02	210302	1,2419E+03	1,1099E-01	3,62	-0,03	-0,03	0,03	0,03
-27	0,0693	0,0763	7,9308E-04	5,2675E-07	0,453	4,5808E+00	1,8276E+02	230447	1,8183E+03	1,0051E-01	4,07	-0,04	-0,03	0,03	0,04
-26	0,0763	0,0839	9,5963E-04	7,7121E-07	0,453	5,7008E+00	2,3867E+02	248712	2,6622E+03	8,9653E-02	4,53	-0,04	-0,04	0,04	0,04
-25	0,0839	0,0923	1,1612E-03	1,1291E-06	0,453	7,0560E+00	3,0787E+02	265141	3,8977E+03	7,8988E-02	4,98	-0,05	-0,04	0,04	0,05
-24	0,0923	0,1015	1,4050E-03	1,6532E-06	0,453	8,6958E+00	3,9315E+02	279822	5,7066E+03	6,8894E-02	5,43	-0,05	-0,05	0,05	0,05
-23	0,1015	0,1117	1,7000E-03	2,4204E-06	0,453	1,0680E+01	4,9789E+02	292868	8,3551E+03	5,9591E-02	5,89	-0,06	-0,05	0,05	0,06
-22	0,1117	0,1228	2,0571E-03	3,5437E-06	0,453	1,3081E+01	6,2617E+02	304402	1,2233E+04	5,1189E-02	6,34	-0,06	-0,06	0,06	0,06
-21	0,1228	0,1351	2,4890E-03	5,1883E-06	0,453	1,5986E+01	7,8295E+02	314558	1,7910E+04	4,3716E-02	6,79	-0,07	-0,06	0,06	0,07
-20	0,1351	0,1486	3,0117E-03	7,5962E-06	0,453	1,9501E+01	9,7419E+02	323465	2,6222E+04	3,7152E-02	7,24	-0,07	-0,07	0,07	0,07
-19	0,1486	0,1635	3,6442E-03	1,1122E-05	0,453	2,3754E+01	1,2071E+03	331252	3,8391E+04	3,1443E-02	7,70	-0,08	-0,07	0,07	0,08
-18	0,1635	0,1799	4,4095E-03	1,6283E-05	0,453	2,8901E+01	1,4906E+03	338040	5,6209E+04	2,6519E-02	8,15	-0,09	-0,08	0,08	0,09
-17	0,1799	0,1978	5,3355E-03	2,3840E-05	0,453	3,5128E+01	1,8351E+03	343939	8,2295E+04	2,2299E-02	8,60	-0,10	-0,09	0,09	0,10


Superestrutura Cônica Vertical Peso Próprio: D/d =1.1, H = 32.60m, D = 30.91m, M =876t
20,00
15,00
10,00
5,00
0,00
-5,00
-10,00
-15,00
-20,00
0,00
5,00
10,00
15,00
20,00
25,00
30,00
35,00
-Di/2 [m]
-di/2 [m]
+di/2 [m]
+Di/2 [m]

Superestrutura Elevador Espacial Xerxes (resiste ao Peso Próprio) Alessandro, 08/jan/2025

CONCLUSÃO

- Esta Superestrutura Cônica Vertical é capaz de resistir ao Peso Próprio, conforme concebida em material AL7475-T7451 para uma altura qualquer; inclusive aquela desejada H = 540000m = 540 km
- Isso se deve ao fato do comprimento para a estabilidade estrutural se manter constante em L_i = 0.456m (vide slides 3 e 4) ou L_i = 0.453m (vide slide 6), a depender do início da iteração (i=1 ou i=-36)
- Apesar diâmetro crescer exponencialmente com a altura, uma Superestrutura Cônica Vertical muito elevada não terá uma área de base nem massa aumentadas, desde que se cresça usando iterações negativas i=-36 (vide slides 6 e7)
- Esta Família de Superestruturas Cônicas Verticais capazes de resistir ao Peso Próprio pode ter aplicações Estéticas, na Arquitetura e na Engenharia Civil
- Esta Família lembra aquela dos troncos de árvores, apresentando semelhanças com estruturas observáveis na natureza
- Para saber mais sobre estruturas esbeltas capazes de resistir ao peso próprio, acesse https://en.wikipedia.org/wiki/Self-buckling
- Para saber mais sobre elevadores espaciais, o conceito que originou este estudo, acesse https://en.wikipedia.org/wiki/Space_elevator
- Li sobre a ideia no The Three-Body Problem, premiado livro de ficção científica https://en.wikipedia.org/wiki/The_Three-Body_Problem_(novel)
- Fique Bem \o/